Lesson Two

What about $crd1°$?

You´ve Probably noticed that despite the various formulas introduced till now you cannot determine the value of $crd1°$ ... Ptolemy has also faced this problem. Without this value it is impossible to move forward and get the remaining chord table ... Ptolemy found a solution that he proved rigorously using geometric arguments.

Theorem 1 deduction with support in GeoGebra

1. Open the link: http://www.geogebratube.org/student/m123345
2. Animate the selector.
3. Observe the table that is being generated.
   a) Check what happens when $\beta = 1°, 30°, 45°, 60°$ e $89°$:
      - compare the values of β with the values of α

      - compare the values of $crd\beta$ with the values of $crd\alpha$;

      - compare the ratios between angles with the ratios between chords.

   b) Does the relationship between angles and chords, previously observed, occur for any value of β?
   c) Write a conjecture for a relation expressing the ratios between angles and their respective chords.

Extension: Prove the result which allows the determination of the $crd1°$.

Application: determining $crd1°$

1. Apply Theorem 1 to $\alpha = 1\frac{1}{2}°$ and $\beta = 1°$. Obtain a lower bound for $crd1°$.

2. Apply Theorem 1 to $\alpha = 1°$ and $\beta = \frac{3}{4}°$. Obtain a upper bound for $crd1°$.

3. Conclude that $crd1° = 1;2,50$ with two sexagesimal places.
4. From the previous value determine $crd\left(\frac{1}{2}°\right)$.

Generating the missing chords

Using the value of $crd1°$, complete the remaining chord table.

1. Open the link: http://www.geogebratube.org/student/m123376

2. Animate angle α and observe the values that are generated in the spreadsheet.

3. Observe the last column of the Excel worksheet. What is the minimum accuracy of the values of Ptolemy?

1. Open the link: http://www.geogebratube.org/student/m123394

2. Starting from the 23 angles of Ptolemy´s table (θ) and the respective ratios between chords and $120 \cdot ((crd(\theta))/120)$, one can get $\sin\left(\frac{\theta}{2}\right)$. In the file Cartesian referential, ordered pairs $\left(\theta, \sin\left(\frac{\theta}{2}\right)\right)$ are marked:

   - select the check boxes to get the sinusoidal and polynomial regressions;

   - select the check boxes to get the difference function between the sine function and each of the regressions constructed.

3. What is in your opinion the better regression? Justify your choice.